Questions 1, 2 and 3 each weigh 1/3. These weights, however, are only indicative for the overall evaluation.

Henrik Jensen
Department of Economics
University of Copenhagen

MONETARY POLICY
SUGGESTED SOLUTIONS TO JUNE 9 EXAM, 2017

QUESTION 1:

Evaluate whether the following statements are true or false. Explain your answers.

(i) In any flex-price model, where both consumption and investment in productive capital is subject to a cash-in-advance constraint, superneutrality prevails such that changes in nominal interest rates have no real effects.

A **False.** The cash-in-advance constraint will effectively make the nominal interest rate a distorting tax on investment and thereby capital accumulation. This means that the steady-state capital stock and output will be affected and superneutrality fails. Higher inflation will be associated with a higher nominal interest rate, and therefore a higher opportunity cost of holding the needed cash. This will increase the cost of investing and lead to lower capital and output in the long run.

(ii) A relatively high coefficient on output in a Taylor-type interest-rate rule is always a sign of an optimizing central bank with a relatively strong preference for output stability.

A **False**. An optimizing central bank will respond to whatever variables that may attain its goals as well as possible. This means that a central bank may well choose to respond to a variable that it has no preference for, as long as the variable provides relevant information. I.e., the central bank engages in intermediate targeting. In the inflation-targeting model of the curriculum, this is exemplified in the special case where the central bank only cares about inflation stability. In that case it is nevertheless optimal to respond to output, as output provides information about inflation. A higher response to output would therefore not reflect any change in preferences, but reflect that the "informational value" of output has increased.

(iii) If the central bank has little control over the broad money supply, the insights from Poole's 1970 model posit that the central bank should adopt a base money operating procedure rather than an interest-rate operating procedure.

A **False.** According to the Poole model, an interest-rate operating procedure is beneficial when money-market shocks are predominant. In that case, using the interest rate as the policy instrument, insulates the economy from such money-market shocks. If one to the basic model adds the feature that the central bank realistically has imperfect control over the broad money stock, and can use base money as an instrument, the argument against monetary operating procedures is further strengthened. Apart from making the economy subject to money-demand/supply shock, one is also making it subject to variations in the money multiplier. Hence, the case for an interest-rate operating procedure is stronger.

QUESTION 2:

Assume a closed economy in discrete time, where households maximize

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, m_t, n_t) \tag{1}$$

with

$$u(c_t, m_t, n_t) \equiv \frac{(c_t m_t)^{1-\Phi}}{1-\Phi} + \frac{(1-n_t)^{1-\eta}}{1-\eta}, \quad \Phi > 0, \quad \eta > 0,$$

subject to the budget constraint

$$f(k_{t-1}, n_t) + \tau_t + (1-\delta) k_{t-1} + \frac{1}{1+\pi_t} m_{t-1} = c_t + k_t + m_t, \tag{2}$$

where c_t is consumption, m_t is real money balances at the end of period t, n_t is labor supply, k_{t-1} is physical capital at the end of period $t-1$, τ_t are monetary transfers, $0 < \delta < 1$ is the depreciation rate of capital, and π_t is the inflation rate. The function f is defined as

$$f(k_{t-1}, n_t) = A k_{t-1}^{\alpha} n_t^{1-\alpha}, \quad 0 < \alpha < 1.$$

(i) Discuss why money may enter the utility function, and describe (2) in detail.

A Here one should mention that it can be a way of modelling the liquidity services of holding money. For example in terms of saved shopping time. For a description of the budget constraint, it suffices to briefly mention the components of total available ressources, as well as their possible usage.

(ii) Derive the relevant first-order conditions for optimal choices of c, m, and n subject to (1) and the definition

$$a_t \equiv \tau_t + \frac{1}{1+\pi_t} m_{t-1} \tag{3}$$

[Hint: Set up the value function $V(a_t, k_{t-1}) = \max_{c_t, m_t, n_t} \{u(c_t, m_t, n_t) + \beta V(a_{t+1}, k_t)\}$ and substitute out k_t and a_{t+1} by use of (2) and (3) respectively.]

Interpret the first-order conditions for c_t, m_t, and n_t intuitively, and show that they can be combined into (along with the expressions for the partial derivatives of the value function):

$$u_m(c_t, m_t, n_t) + \frac{\beta}{1+\pi_{t+1}} u_c(c_{t+1}, m_{t+1}, n_{t+1}) = u_c(c_t, m_t, n_t), \tag{4}$$

$$u_c(c_t, m_t, n_t) = \beta R_t u_c(c_{t+1}, m_{t+1}, n_{t+1}), \tag{5}$$

$$-u_n(c_t, m_t, n_t) = u_c(c_t, m_t, n_t) f_n(k_{t-1}, n_t), \tag{6}$$

where $R_t \equiv f_k(k_t, n_{t+1}) + 1 - \delta$ is the gross real interest rate, which equals $(1+i_t)/(1+\pi_{t+1})$, with i_t being the nominal interest rate.

A From

$$V(a_t, k_{t-1}) = \max_{c_t, m_t, n_t} \{u(c_t, m_t, n_t) + \beta V(a_{t+1}, k_t)\},$$

we derive the first-order conditions for c_t, m_t, n_t, respectively:

$$\begin{aligned}
u_c(c_t, m_t, n_t) + \beta V_a(a_{t+1}, k_t) \frac{\partial a_{t+1}}{\partial c_t} + \beta V_k(a_{t+1}, k_t) \frac{\partial k_t}{\partial c_t} &= 0, \\
u_m(c_t, m_t, n_t) + \beta V_a(a_{t+1}, k_t) \frac{\partial a_{t+1}}{\partial m_t} + \beta V_k(a_{t+1}, k_t) \frac{\partial k_t}{\partial m_t} &= 0, \\
u_n(c_t, m_t, n_t) + \beta V_a(a_{t+1}, k_t) \frac{\partial a_{t+1}}{\partial n_t} + \beta V_k(a_{t+1}, k_t) \frac{\partial k_t}{\partial n_t} &= 0,
\end{aligned}$$

which by use of

$$\begin{aligned}
k_t &= f(k_{t-1}, n_t) + \tau_t + (1-\delta) k_{t-1} + \frac{1}{1+\pi_t} m_{t-1} - c_t - m_t \\
a_{t+1} &= \tau_{t+1} + \frac{1}{1+\pi_{t+1}} m_t
\end{aligned}$$

[from (2) and (3), respectively] become

$$\begin{aligned} u_c(c_t,m_t,n_t) - \beta V_k(a_{t+1},k_t) &= 0, \\ u_m(c_t,m_t,n_t) + \beta V_a(a_{t+1},k_t)\frac{1}{1+\pi_{t+1}} - \beta V_k(a_{t+1},k_t) &= 0, \\ u_n(c_t,m_t,n_t) + \beta V_k(a_{t+1},k_t) f_n(k_{t-1},n_t) &= 0. \end{aligned}$$

Slightly rewritten, this becomes

$$u_c(c_t,m_t,n_t) = \beta V_k(a_{t+1},k_t), \tag{*}$$

$$u_m(c_t,m_t,n_t) + \beta V_a(a_{t+1},k_t)\frac{1}{1+\pi_{t+1}} = \beta V_k(a_{t+1},k_t), \tag{**}$$

$$-u_n(c_t,m_t,n_t) = \beta V_k(a_{t+1},k_t) f_n(k_{t-1},n_t). \tag{***}$$

Equation (*) shows that consumption is chosen such that its marginal gain, in terms of current utility, equals its marginal loss which is the discounted future marginal value of capital. Equation (**) shows essentially the same for the choice of real money, but with an additional marginal gain in terms of the discounted future marginal gain of real monetary resources. Finally, (***) shows that labor supply is chosen so the marginal cost of labor, in terms of higher disutility, equals the marginal gain in terms of the discounted future marginal value of capital times the extra resources created by higher labor.

We can now find the partial derivatives of the value function by exploiting that

$$V(a_t,k_{t-1}) = u(c_t,m_t,n_t) + \beta V(a_{t+1},k_t)$$

with a_{t+1} and k_t given by (3) and (2), respectively, *and* that c_t, m_t and n_t are chosen optimally. The latter enables us by the Envelope Theorem to ignore any effects of a_t and k_{t-1} on c_t, m_t and n_t. The derivatives then follow as

$$\begin{aligned} V_a(a_t,k_{t-1}) &= \beta\left[V_a(a_{t+1},k_t)\frac{\partial a_{t+1}}{\partial a_t} + V_k(a_{t+1},k_t)\frac{\partial k_t}{\partial a_t}\right], \\ V_k(a_t,k_{t-1}) &= \beta\left[V_k(a_{t+1},k_t)\frac{\partial a_{t+1}}{\partial k_{t-1}} + V_k(a_{t+1},k_t)\frac{\partial k_t}{\partial k_{t-1}}\right]; \end{aligned}$$

or,

$$V_a(a_t,k_{t-1}) = \beta V_k(a_{t+1},k_t), \tag{****}$$

$$V_k(a_t,k_{t-1}) = \beta V_k(a_{t+1},k_t)\left[f_k(k_{t-1},n_t) + 1 - \delta\right]; \tag{*****}$$

where the first equality follows as $\partial a_{t+1}/\partial a_t = 0$ and $\partial k_t/\partial a_t = 1$ as $a_t = \tau_t + m_{t-1}/(1+\pi_t)$ is inserted into the budget constraint. Likewise, the second equality uses $\partial a_{t+1}/\partial k_{t-1} = 0$ and $\partial k_t/\partial k_{t-1} = f_k(k_{t-1}, n_t) + 1 - \delta$.
We can now proceed to find (4), (5) and (6). First note, that (6) follows immediately by a combination of (*) and (***). To find (5), forward (*****) to get

$$V_k(a_{t+1}, k_t) = \beta V_k(a_{t+2}, k_{t+1})\left[f_k(k_t, n_{t+1}) + 1 - \delta\right],$$

and use (*) and the definition of R_t to obtain

$$\begin{aligned} \beta^{-1} u_c(c_t, m_t, n_t) &= R_t u_c(c_{t+1}, m_{t+1}, n_{t+1}), \\ u_c(c_t, m_t, n_t) &= \beta R_t u_c(c_{t+1}, m_{t+1}, n_{t+1}), \end{aligned}$$

which is (5). Finally (4) is recovered from (**) where (****) is used to substitute out $V_a(a_{t+1}, k_t)$:

$$u_m(c_t, m_t, n_t) + \beta^2 V_k(a_{t+2}, k_{t+1}) \frac{1}{1+\pi_{t+1}} = \beta V_k(a_{t+1}, k_t).$$

Finally, (*) is used to substitute out V_k:

$$u_m(c_t, m_t, n_t) + u_c(c_{t+1}, m_{t+1}, n_{t+1}) \frac{\beta}{1+\pi_{t+1}} = u_c(c_t, m_t, n_t),$$

which is (4).

(iii) Using the specific functional forms for u and f, examine the properties of the steady state using (4), (5), and (6) together with the national account identity $c^{ss} = A {k^{ss}}^{\alpha} {n^{ss}}^{1-\alpha} - \delta k^{ss}$. Discuss under which circumstances the model exhibits superneutrality, and discuss whether the correlation between output and inflation is unambiguous. [Note: You are not required to explicitly solve for all variables, but to use the equations as input to your arguments.]

A With the given functional forms,

$$\begin{aligned} u_c(c^{ss}, m^{ss}, n^{ss}) &= (m^{ss})^{1-\Phi} (c^{ss})^{-\Phi}, \\ u_m(c^{ss}, m^{ss}, n^{ss}) &= (c^{ss})^{1-\Phi} (m^{ss})^{-\Phi}, \\ -u_n(c^{ss}, m^{ss}, n^{ss}) &= (1-n_t)^{-\eta}, \\ f_n(k^{ss}, n^{ss}) &= A(1-\alpha)(k^{ss}/n^{ss})^{\alpha}, \\ f_k(k^{ss}, n^{ss}) &= A\alpha(n^{ss}/k^{ss})^{1-\alpha}, \end{aligned}$$

equations (4), (5) and (6) become

$$\left(c^{ss}\right)^{1-\Phi}\left(m^{ss}\right)^{-\Phi}+\frac{\beta}{1+\pi^{ss}}\left(m^{ss}\right)^{1-\Phi}\left(c^{ss}\right)^{-\Phi} = \left(m^{ss}\right)^{1-\Phi}\left(c^{ss}\right)^{-\Phi}, \tag{4'}$$

$$\beta^{-1} = R^{ss}=A\alpha\left(n^{ss}/k^{ss}\right)^{1-\alpha}+1-\delta, \tag{5'}$$

$$\left(1-n_t\right)^{-\eta} = \left(m^{ss}\right)^{1-\Phi}\left(c^{ss}\right)^{-\Phi}A\left(1-\alpha\right)\left(k^{ss}/n^{ss}\right)^{\alpha}. \tag{6'}$$

Here it can be observed from (5') that the real interest rate in steady state is exclusively determined by the subjective discount factor. With the Cobb-Douglas production function this means that the capital-labor ratio is independent of monetary factors. From (4') we readily get

$$c^{ss}+\frac{\beta}{1+\pi^{ss}}m^{ss}=m^{ss},$$

or

$$\begin{aligned} m^{ss}\left(1-\frac{\beta}{1+\pi^{ss}}\right) &= c^{ss} \\ m^{ss}\left(1-\frac{1}{R^{ss}\left(1+\pi^{ss}\right)}\right) &= c^{ss} \\ m^{ss}\left(1-\frac{1}{1+i^{ss}}\right) &= c^{ss} \end{aligned}$$

where the last line uses the definition of the nominal interest rate. Hence, we get a simple money demand relationship:

$$m^{ss}=c^{ss}\frac{1+i^{ss}}{i^{ss}},$$

with real money depending negatively on the nominal interest rate—and since the real interest rate is constant, negatively on the inflation rate.

Real money is therefore affected by monetary policy, and by inspection of (6') we see that in the special case of $\Phi=1$, different values of the money stock will not affect labor supply. As the capital to labor ratio is fixed, capital is unchanged and output and consumption will not change either as seen from the national account. This means that in this special case, monetary policy is superneutral. With, $\Phi\neq 1$ on the other hand, changes in inflation and the nominal interest rate will affect labor supply, as the marginal utility of consumption, $\left(m^{ss}\right)^{1-\Phi}\left(c^{ss}\right)^{-\Phi}$, is affected. E.g., if $\Phi>1$, higher inflation reduces m^{ss} and thereby increases the marginal utility of consumption. This will induce households to choose more consumption over leisure, and labour supply goes up. Output will likely increase. For $\Phi < 1$ the effect will be the opposite. I.e., for $\Phi \neq 1$ superneutrality fails in the model, and the direction by which monetary policy changes affect output, depends on $\Phi \lesseqgtr 1$.

QUESTION 3:

Consider the following New-Keynesian log-linear model of a closed economy:

$$
\begin{aligned}
x_t &= \mathrm{E}_t x_{t+1} - \left(\widehat{i}_t - \mathrm{E}_t \pi_{t+1}\right) + u_t, && (1)\\
\pi_t &= \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t, \qquad 0 < \beta < 1, \quad \kappa > 0, && (2)\\
\widehat{i}_t &= \phi_\pi \pi_t + \phi_x x_t^o, \qquad \phi_\pi > 1, \quad \phi_x > 0, && (3)\\
x_t^o &= x_t + e_t, && (4)
\end{aligned}
$$

where x_t is the output gap, $\widehat{i}_t$ is the nominal interest rate's deviation from steady state, and π_t is goods-price inflation, u_t is mean-zero i.i.d. shock. E_t is the rational-expectations operator conditional upon all information up to and including period t. The variable x_t^o is the observed output gap, which is an imperfect measure of the actual output gap: e_t is a mean-zero i.i.d. shock.

(i) Derive the solutions for x_t and π_t. [Hint: Conjecture that the solutions are linear functions of u_t and e_t, and use the method of undetermined coefficients.] Explain carefully how the shocks are transmitted onto the variables.

A Using the hint, we conjecture that

$$
\begin{aligned}
x_t &= Au_t - Be_t,\\
\pi_t &= Cu_t - De_t,
\end{aligned}
$$

where A, B, C, and D are coefficients to be determined. Since both shocks have no persistence and zero mean, this conjecture implies

$$\mathrm{E}_t x_{t+1} = \mathrm{E}_t \pi_{t+1} = 0.$$

Note that we can substitute (4) into (3) into (1) to get

$$
\begin{aligned}
x_t &= \mathrm{E}_t x_{t+1} - \phi_\pi \pi_t - \phi_x \left(x_t + e_t\right) + \mathrm{E}_t \pi_{t+1} + u_t\\
\pi_t &= \beta \mathrm{E}_t \pi_{t+1} + \kappa x_t.
\end{aligned}
$$

Now substitute in the conjectures and their expectations to get

$$
\begin{aligned}
Au_t - Be_t &= -\phi_\pi \left(Cu_t - De_t\right) - \phi_x \left(Au_t - Be_t + e_t\right) + u_t\\
Cu_t - De_t &= \kappa \left(Au_t - Be_t\right).
\end{aligned}
$$

These are linear equations in the shocks thereby verifying our conjecture. As the two equations must hold for all values of the shock, we have that the following restriction must be satisfied in a solution

$$
\begin{aligned}
A &= -\phi_\pi C - \phi_x A + 1, \\
-B &= \phi_\pi D + \phi_x (B - 1), \\
C &= \kappa A, \\
-D &= -\kappa B.
\end{aligned}
$$

These four equations determine the solutions for A, B, C, and D. Inserting the last two equations in the first two gives

$$
\begin{aligned}
A &= -\phi_\pi \kappa A - \phi_x A + 1, \\
-B &= \phi_\pi \kappa B + \phi_x (B - 1),
\end{aligned}
$$

and thus

$$
\begin{aligned}
A (1 + \phi_\pi \kappa + \phi_x) &= 1, \\
-B (1 + \phi_\pi \kappa + \phi_x) &= -\phi_x,
\end{aligned}
$$

$$
\begin{aligned}
A &= \frac{1}{1 + \phi_\pi \kappa + \phi_x}, \\
B &= \frac{\phi_x}{1 + \phi_\pi \kappa + \phi_x},
\end{aligned}
$$

leading to

$$
\begin{aligned}
C &= \frac{\kappa}{1 + \phi_\pi \kappa + \phi_x}, \\
D &= \frac{\phi_x \kappa}{1 + \phi_\pi \kappa + \phi_x}.
\end{aligned}
$$

Hence, the solutions for x_t and π_t are

$$
x_t = \frac{1}{1 + \phi_\pi \kappa + \phi_x} u_t - \frac{\phi_x}{1 + \phi_\pi \kappa + \phi_x} e_t, \qquad (*)
$$

$$
\pi_t = \frac{\kappa}{1 + \phi_\pi \kappa + \phi_x} u_t - \frac{\phi_x \kappa}{1 + \phi_\pi \kappa + \phi_x} e_t. \qquad (**)
$$

We see from (*) that a positive realization of u_t leads to increased demand and thus output gap by (1). This feeds into inflation via (2). With the assumed policy rule we see, however, from (*) and (**) that the effects on x_t and π_t are dampened, and the more, the higher are the values of coefficients ϕ_π and ϕ_x. This is because the associated increase in the nominal interest rate increases the real interest rate and thus reduces demand and the direct effect of the shock. A positive realization of e_t leads to a higher observed output gap. This leads the central bank to raise the nominal interest rate (as $\phi_x > 0$), which causes demand to fall and thereby the actual output gap and inflation to fall.

(ii) Assume that stabilizing the output gap, x_t and π_t is preferable. Discuss why this is an assumption often used in this type of model.

A In this type of model, the underlying structure posits a proportional relationship between the output gap and real marginal costs. Variations in real marginal costs are welfare costly under sticky prices as this will reflect deviations from efficiency (achieved when prices equal nominal marginal costs). Hence, fluctuations in the output gap is also costly. Inflation is costly under sticky prices of the Calvo form, at it causes a misallocation in production across different types of goods in the economy. This is costly for consumers who prefer an evenly distributed consumption bundle.

(iii) Evaluate formally whether stabilizing x_t and π_t perfectly *at the same time,* is possible in the model by appropriate choices of ϕ_π and ϕ_x. Discuss.

A From (*) and (**) one can see that any implications of the imperfect measurement of the output gap on the economy can be eliminated by $\phi_x = 0$. In that case the central bank completely ignores observed changes in the output gap. By doing this, the inevitable measurement errors in the observation will not be transmitted onto the economy at all. This leaves the question of whether the effects of the u_t shock can be neutralized entirely on both inflation and output? Inspecting (*) and (**) reveals that $\phi_\pi \to \infty$ will imply $\partial x_t / \partial u_t = \partial \pi_t / \partial u_t = 0$. Hence, setting policy coefficients according to $\phi_\pi \to \infty$, and $\phi_x \to 0$ will stabilize the output gap and inflation completely. One may say that the "divine coincidence" applies. (One may note that in that case, the nominal interest rate

$$\widehat{i}_t = \phi_\pi \left(\frac{\kappa}{1 + \phi_\pi \kappa + \phi_x} u_t - \frac{\phi_x \kappa}{1 + \phi_\pi \kappa + \phi_x} e_t \right) + \phi_x x_t^o$$

is

$$\left. \widehat{i}_t \right|_{\phi_x \to 0} = \phi_\pi \frac{\kappa}{1 + \phi_\pi \kappa} u_t$$

and thus

$$\lim_{\phi_\pi \to \infty} \left[\left. \widehat{i}_t \right|_{\phi_x \to 0} \right] = u_t.$$

I.e., the nominal interest rate tracks perfectly the "natural rate of interest". Also one can note, that $\phi_\pi \to \infty$ is sufficient for the attainment of both goals.)